Pergamon

Int. J. Heat Mass Transfer. Vol. 39, No. 17, pp. 3725–3732, 1996

0017–9310/96 $15.00+0.00

0017–9310(95)00359–2

The effect of liquid encapsulation on the Marangoni convection in a liquid column under microgravity condition

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(*Received* 15 *May* 1995)

Abstract—The present paper is devoted to the investigation of the effect of liquid encapsulation on the Marangoni convection in its inner columnar liquid layer under a microgravity condition. The asymptotic solution of the mathematic model established in the paper was obtained for the immiscible axisymmetric coaxial liquid columns contained between planar faces. Numerical simulation based on the asymptotic solution was carried out. The main influence factors on the process were discussed in detail. It is found that proper selection of the related parameters will result in a remarkable reduction of the Marangoni convective flow. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

It is well known that thermocapillary (or Marangoni) convection induced by the gradient of surface tension as a function of temperature plays an important role under a microgravity condition where the buoyance-driven convection is negligible. In order to grow high quality large sized crystals a technique called liquid encapsulation was developed in which a two layer liquid system will be involved. Compared with the case of single layer, the two layer system is much more complicated because the continuity conditions must be satisfied at the interface between two immiscible fluids. In refs. [1–6] Marangoni convection in multilayer system was studied from different angles. However, a more thorough investigation on the interaction between two liquid layers is still an open problem which is urgent, not only from viewpoint of theoretical study, but also from the practical applications.

PHYSICAL AND MATHEMATICAL MODEL

Consider two immiscible axisymmetric coaxial liquid columns contained between planar faces with a distance L apart, as shown in Fig. 1. Let R_1 and R_2 be the radius of the inner and the outer liquid columnar surface. The liquid is an incompressible Newtonian fluid of density ρ_i, thermal diffusivity κ_i, thermal conductivity λ_i, kinematic viscosity μ_i and dynamic viscosity v_i ($i = 1,2$ for liquid 1 and liquid 2, respectively). The liquid–liquid interface 1 and liquid–gas interface 2 are assumed to be undeformable. The endwalls at $z = \pm L/2$ are maintained at temperature T_c and T_h. Let β be a measure of the temperature gradient along the liquid–gas interface 2. The outer liquid 2 is surrounded by a passive gas having negligible density and viscosity. The interfaces 1 and 2 possess surface tension σ_1 and σ_2, respectively. The law of surface tension vs temperature is assumed to be linear

$$\sigma_i = \sigma_{0i} - \gamma_i(T_i - T_0)$$
$$\text{with } T_0 = (1/2)(T_h + T_c)(i = 1,2) \quad (1)$$

in which σ_{01} and σ_{02} is the surface tension of interface 1 and interface 2 at temperature T_0, γ_i is the variation rate of surface tension with temperature T, where $i = 1$ for the liquid–liquid interface and $i = 2$ for the liquid–gas interface. Gravity is assumed to be absent.

Under conditions assumed before, the steady axisymmetric motion of liquid i is governed by the following equations:

$$\frac{1}{r}(ru_i)_r + w_{iz} = 0 \quad (2a)$$

$$u_i u_{ir} + w_i u_{iz} = -\frac{1}{\rho_i} p_{ir} + v_i(\nabla^2 - 1/r^2)u_i \quad (2b)$$

$$u_i w_{ir} + w_i w_{iz} = -\frac{1}{\rho_i} p_{iz} + v_i \nabla^2 w_i \quad (2c)$$

$$u_i T_{ir} + w_i T_{iz} = \kappa_i \nabla^2 T_i \quad (2d)$$

here, the Laplacian operator is shown by

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$$

p_i ($i = 1,2$) is the liquid pressure for liquid 1 and 2, respectively; u_i and w_i are the velocity components of

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NOMENCLATURE

A	aspect ratio, R_2/L
Bi	Biot number, hR_2/λ_2
C_{ij}	integrate constant
h	heat transfer coefficient
L	length of liquid bridge
Ma	Marangoni number, $\gamma_2\beta R_2^2/\mu_2\kappa_2$
R_1	inner layer radius
R_2	outer layer radius
Re	Reynolds number, $\gamma_2\beta R_2^2/\mu_2\nu_2$
T_a	gas temperature
T_c	temperature at cold planar face
T_h	temperature at hot planar face
w^*	reference axial velocity
a	radius ratio of inner and outer liquid columns
p	dimensionless pressure
r	dimensionless radial coordinate
u_i	dimensionless radial velocity component for ith layer
w_i	dimensionless axial velocity component for ith layer
z	dimensionless axial coordinate.

Greek symbols

β	temperature gradient along z coordinate
γ	surface-tension temperature coefficient
κ	thermal diffusivity
λ	thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
σ	surface tension
Φ	any physical variable.

Subscript

s	for single layer.

Superscript

*	physical property ratio of liquid 1 to liquid 2.

liquid i in the directions (r, z), respectively, and the cylindrical coordinate system is shown in Fig. 1, subscripts denote the partial differentiation.

Equations (2a–d) are subject to the following boundary conditions:

No-slip and isothermal conditions considered at the rigid endwalls

$$z = \frac{1}{2}L: \quad u_i = w_i = 0 \quad T_i = T_c \tag{3a}$$

$$z = -\frac{1}{2}L: \quad u_i = w_i = 0 \quad T_i = T_h. \tag{3b}$$

On the liquid–liquid interface 1, $r = R_1$

$$u_1 = 0 \tag{3c}$$

$$w_1 = w_2 \tag{3d}$$

$$T_1 = T_2 \tag{3e}$$

$$\lambda_1 \frac{\partial T_1}{\partial r} = \lambda_2 \frac{\partial T_2}{\partial r} \tag{3f}$$

$$\mu_1 \frac{\partial w_1}{\partial r} - \mu_2 \frac{\partial w_2}{\partial r} = -\gamma_1 \frac{\partial T}{\partial z}. \tag{3g}$$

On the liquid–gas interface $r = R_2$

$$u_2 = 0 \tag{3h}$$

$$\mu_2 \frac{\partial w_2}{\partial r} = -\gamma_2 \frac{\partial T_2}{\partial z} \tag{3i}$$

$$-\lambda_2 \frac{\partial T_2}{\partial r} = h[T_2 - T_a(z)]. \tag{3j}$$

The thermal condition (3j) presumes that the air temperature $T_a(z)$ is known and that the heat transported at the liquid–gas interface can be described by using a heat-transfer coefficient h.

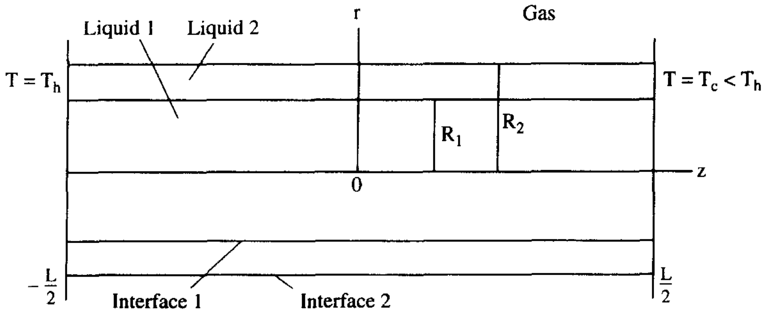

Fig. 1. Physical model.

At both ends, $x = \pm(L/2)$,

$$T_a\left(\frac{1}{2}L\right) = T_c \quad T_a\left(-\frac{1}{2}L\right) = T_h. \qquad (3k,l)$$

The physical quantities are bounded at $r = 0$ as

$$r = 0: \quad u_1, w_1, p_1, T_1 < \infty. \qquad (3m)$$

The velocity field must also satisfy the conditions:

$$\int_0^{R_1} w_1 r\,dr = 0 \qquad (3n)$$

$$\int_{R_1}^{R_2} w_2 r\,dr = 0 \qquad (3p)$$

which follows from the fact that no net mass flow into or out of the liquid column exists.

Introduce the following primed quantities:

$$z = Lz' \quad r = R_2 r' \quad w_i = w_* w_i' \quad u_i = A w_* u_i'$$

$$p_i = \frac{\mu_2 w_* L}{R_2^2} p_i' \quad T_i - T_0 = \beta L T_i' \qquad (4)$$

where A is the aspect ratio, i.e.

$$A = \frac{R_2}{L}$$

and the characteristic velocity w_* derived from the balance of condition (3i) as

$$w_* = \frac{\gamma_2 \beta R_2}{\mu_2}.$$

Then, with the primes dropped conventionally, the dimensionless equations can be written as:

$$\frac{1}{r}(ru_1)_r + w_{1z} = 0 \qquad (5a)$$

$$A^3 Re(u_1 u_{1r} + w_1 u_{1z}) = -\frac{1}{\rho^*} p_{1r} + \nu^* A^2\left(u_{1rr} + \frac{u_{1r}}{r} - \frac{u_1}{r^2} + A^2 u_{1zz}\right) \qquad (5b)$$

$$A Re(u_1 w_{1r} + w_1 w_{1z}) = -\frac{1}{\rho^*} p_{1z} + \nu^*\left(w_{1rr} + \frac{w_{1r}}{r} + A^2 w_{1zz}\right) \qquad (5c)$$

$$A Ma(u_1 T_{1r} + w_1 T_{1z}) = \kappa^*\left(T_{1rr} + \frac{T_{1r}}{r} + A^2 T_{1zz}\right)$$

$$(0 < r < a) \qquad (5d)$$

$$\frac{1}{r}(ru_2)_r + w_{2z} = 0 \qquad (5e)$$

$$A^3 Re(u_2 u_{2r} + w_2 u_{2z}) = -p_{2r} + A^2(u_{2rr} + \frac{u_{2r}}{r} - \frac{u_2}{r^2} + A^2 u_{2zz}) \qquad (5f)$$

$$A Re(u_2 w_{2r} + w_2 w_{2z}) = -p_{2z} + \left(w_{2rr} + \frac{w_{2r}}{r} + A^2 W_{2zz}\right) \qquad (5g)$$

$$A Ma(u_2 T_{2r} + w_2 T_{2z}) = T_{2rr} + \frac{T_{2r}}{r} + A^2 T_{2zz}$$

$$(a < r < 1) \qquad (5h)$$

$$z = \pm\frac{1}{2} \quad u_1 = w_1 = u_2 = w_2 = 0 \qquad (6a\text{–}d)$$

$$T_1 = T_2 = \mp\frac{1}{2} \qquad (6e,f)$$

$$r = a \quad u_1 = 0 \qquad (6g)$$

$$w_1 = w_2 \qquad (6h)$$

$$T_1 = T_2 \qquad (6i)$$

$$\lambda^* \frac{\partial T_1}{\partial r} = \frac{\partial T_2}{\partial r} \qquad (6j)$$

$$\frac{\partial w_1}{\partial r} - \frac{1}{\mu^*}\frac{\partial w_2}{\partial r} = -\frac{\gamma^*}{\mu^*}\frac{\partial T}{\partial z} \qquad (6k)$$

$$r = 1 \quad u_2 = 0 \qquad (6l)$$

$$\frac{\partial w_2}{\partial r} = -\frac{\partial T_2}{\partial z} \qquad (6m)$$

$$-\frac{\partial T_2}{\partial r} = Bi(T_2 - T_a) \qquad (6n)$$

$$r = 0 \quad u_1, w_1, p_1, T_1 < \infty \qquad (6p)$$

$$\int_0^a w_1 r\,dr = 0 \qquad (6q)$$

$$\int_a^1 w_2 r\,dr = 0. \qquad (6r)$$

The dimensionless numbers in equations (5) are defined as follows:

Reynolds number Re

$$Re = \frac{w_* R_2}{\nu_2} = \frac{\gamma_2 \beta R_2^2}{\mu_2 \nu_2}.$$

Marangoni number Ma

$$Ma = \frac{w_* R_2}{\kappa_2} = \frac{\gamma_2 \beta R_2^2}{\mu_2 \kappa_2}.$$

Biot number Bi

$$Bi = \frac{h R_2}{\lambda_2}$$

and a is the radius ratio of liquid 1 to liquid 2

$$a = \frac{R_1}{R_2}.$$

Superscript stands for the physical property ratio of liquid 1 to liquid 2. (e.g. $\mu^* = \mu_1/\mu_2$), and symbol γ^* is defined as $\gamma^* = \gamma_1/\gamma_2$.

ASYMPTOTIC SOLUTION

With restrictions of Ma and Re by

$$Ma, Re = o(A^{-1})$$

we write all unknowns Φ in powers of A as follows:

$$\Phi = \Phi_0 + A\Phi_1 + O(A^2). \tag{7}$$

By substituting equation (7) into equations (5) and (6) and posing the limitation $A \to 0$, we obtain the following system of equations at the leading order as:

$$\frac{1}{r}(ru_{01})_r + w_{01z} = 0 \tag{8a}$$

$$p_{01r} = 0 \tag{8b}$$

$$-\frac{1}{\mu^*}p_{01z} + w_{01rr} + \frac{1}{r}w_{01r} = 0 \tag{8c}$$

$$T_{01rr} + \frac{1}{r}T_{01r} = 0 \tag{8d}$$

$$\frac{1}{r}(ru_{02})_r + w_{02z} = 0 \tag{8e}$$

$$p_{02r} = 0 \tag{8f}$$

$$-p_{02z} + w_{02rr} + \frac{1}{r}w_{02r} = 0 \tag{8g}$$

$$T_{02rr} + \frac{1}{r}T_{02r} = 0 \tag{8h}$$

$$r = a: \quad u_{01} = 0 \tag{9a}$$

$$w_{01} = w_{02} \tag{9b}$$

$$T_{01} = T_{02} = T_0 \tag{9c}$$

$$\lambda^* \frac{\partial T_{01}}{\partial r} = \frac{\partial T_{02}}{\partial r} \tag{9d}$$

$$\frac{\partial w_{01}}{\partial r} - \frac{1}{\mu^*}\frac{\partial w_{02}}{\partial r} = -\frac{\gamma^*}{\mu^*}\frac{\partial T_0}{\partial z} \tag{9e}$$

$$r = 1: \quad u_{02} = 0 \tag{9f}$$

$$\frac{\partial w_{02}}{\partial r} = -\frac{\partial T_{02}}{\partial z} \tag{9g}$$

$$-\frac{\partial T_{02}}{\partial r} = Bi[T_{02} - T_a(z)] \tag{9h}$$

$$r = 0: \quad u_{01}, w_{01}, p_{01}, T_{01} < \infty \tag{9i–l}$$

$$\int_0^a w_{01} r\, \mathrm{d}r = 0 \tag{9m}$$

$$\int_a^1 w_{02} r\, \mathrm{d}r = 0. \tag{9n}$$

From equations (8) and the conditions (9) we obtain the leading-order solution to express the velocity components. Here primes denote the differentiation with respect to z.

Finally, we obtain the expressions of the velocity components u_{01}, w_{01}, u_{02} and w_{02} at leading order as a function of parameters μ^*, γ^* and a for $0 < a < 1$ as:

$$u_{01} = \frac{1}{8\mu^*\left[a^4 - \frac{(a^2-1)^2}{\mu^*} - 4a^2 + 4\ln a + 3\right]}$$

$$\times \Big\{[a^4 - 4a^2 \ln a - 1 - (a^5 - 4a^3 + 4a\ln a + 3a)\gamma^*]r$$

$$+ \frac{1}{a^2}[-a^4 + 4a^2 \ln a + 1 + (a^5 - 4a^3$$

$$+ 4a \ln a + 3a)\gamma^*]r^3\Big\}T_a''(z) \tag{10a}$$

$$w_{01} = \frac{1}{4\mu^*\left[a^4 - \frac{(a^2-1)^2}{\mu^*} - 4a^2 + 4\ln a + 3\right]}$$

$$\times \Big\{[-a^4 + 4a^2 \ln a + 1 + (a^5 - 4a^3 + 4a\ln a + 3a)\gamma^*]$$

$$+ \frac{2}{a^2}[a^4 - 4a^2 \ln a - 1 - (a^5 - 4a^3$$

$$+ 4a \ln a + 3a)\gamma^*]r^2\Big\}T_a' \tag{10b}$$

$$u_{02} = \frac{1}{2\left[a^4 - \frac{(a^2-1)^2}{\mu^*} - 4a^2 + 4\ln a + 3\right]}$$

$$\times \Big\{\Big[-a^4 \ln a + \frac{a^4}{\mu^*}\ln a + \frac{1}{2}a^4 - \frac{1}{4}\frac{a^4}{\mu^*} - \ln a$$

$$+ \frac{1}{4\mu^*} - \frac{1}{2} + \left(-\frac{a^5}{4\mu^*} + \frac{a^3}{\mu^*}\ln a + \frac{a}{4\mu^*}\right)\gamma^*\Big]r$$

$$+ \Big[a^4 \ln a - \frac{a^4}{\mu^*}\ln a + \frac{a^4}{4\mu^*} - \frac{1}{2}a^4 + \frac{1}{2}a^2 - \frac{a^2}{4\mu^*}$$

$$+ \left(\frac{a^5}{4\mu^*} - \frac{a^3}{\mu^*}\ln a - \frac{a^3}{4\mu^*}\right)\gamma^*\Big]\frac{1}{r}$$

$$+ \left[a^4 - 2a^2 - \frac{a(a^2-1)(a+\gamma^*)}{\mu^*} + 1\right] r \ln r$$

$$+ \frac{1}{4}\Big[2a^2 - \frac{a^2}{\mu^*} - 4\ln a - 2 + \frac{1}{\mu^*} - \Big(\frac{a^3}{\mu^*}$$

$$- \frac{a}{\mu^*}\Big)\gamma^*\Big]r^3\Big\}T_a'' \tag{10c}$$

$$w_{02} = \frac{1}{a^4 - \frac{(a^2-1)^2}{\mu^*} - 4a^2 + 4\ln a + 3}\Big\{a^4 \ln a$$

$$-\frac{a^4}{\mu^*}\ln a-a^4+\frac{3}{4}\frac{a^4}{\mu^*}+a^2-\frac{a^2}{2\mu^*}+\ln a-\frac{1}{4\mu^*}+\frac{1}{\mu^*}$$
$$\times\left(\frac{a^5}{4}-a^3\ln a+\frac{a^3}{2}-\frac{3a}{4}\right)\gamma^*-\left[a^4-2a^2\right.$$
$$\left.+\frac{a(1-a^2)(a+\gamma^*)}{\mu^*}+1\right]\ln r+\frac{1}{2}\left[2a^2-\frac{a^2}{\mu^*}-4\ln a\right.$$
$$\left.\left.-2+\frac{1}{\mu^*}-\frac{1}{\mu^*}(a^3-a)\gamma^*\right]r^2\right\}T'_a. \quad (10d)$$

Here primes denote the differentiation with respect to z.

The flow-induced temperature distribution can be expressed as:

$$T_1\approx T_{01}+AT_{11}=T_a+AMa\times\left[\frac{C_{11}^{0a}}{4\kappa^*}\left(1-\frac{1}{2a^2}r^2\right)+C_{12}^{1T}\right][T'_a(z)]^2 \quad (11a)$$

$$T_2\approx T_{02}+AT_{12}=T_a+AMa\left[\frac{C_{21}^{0a}}{4}r^2+\frac{C_{22}^{0a}}{4}(\ln r-1)r^2+\frac{C_{23}^{0a}}{16}r^4+C_{21}^{1T}\ln r+C_{22}^{1T}\right][T'_a(z)]^2 \quad (11b)$$

where

$$C_{11}^{0a}=\frac{-a^4+4a^2\ln a+1+(a^5-4a^3+4a\ln a+3a)\gamma^*}{4\mu^*\left[a^4-\frac{(a^2-1)^2}{\mu^*}-4a^2+4\ln a+3\right]} \quad (11c)$$

$$C_{21}^{0a}=\frac{a^4\ln a-\frac{a^4}{\mu^*}\ln a-\frac{3}{4}\frac{a^4}{\mu^*}-a^4+a^2-\frac{a^2}{2\mu^*}+\ln a-\frac{1}{4\mu^*}+\frac{1}{\mu^*}\left(\frac{a^5}{4}-a^3\ln a+\frac{a^3}{2}-\frac{3a}{4}\right)\gamma^*}{a^4-\frac{(a^2-1)^2}{\mu^*}-4a^2+4\ln a+3} \quad (11d)$$

$$C_{22}^{0a}=\frac{-a^4+2a^2+\frac{a(a^2-1)(a+\gamma^*)}{\mu^*}}{a^4-\frac{(a^2-1)^2}{\mu^*}-4a^2+4\ln a+3} \quad (11e)$$

$$C_{23}^{0a}=\frac{\frac{1}{2}\left[2a^2-\frac{a^2}{\mu^*}-4a\ln a-2+\frac{1}{\mu^*}-\frac{1}{\mu^*}(a^3-a)\gamma^*\right]}{a^4-\frac{(a^2-1)^2}{\mu^*}-4a^2+4\ln a+3} \quad (11f)$$

$$C_{12}^{1T}=-\frac{a^2}{8\kappa^*}C_{11}^{0a}+\frac{a^2}{4}C_{21}^{0a}+\frac{a^2}{4}(\ln a-1)C_{22}^{0a}+\frac{a^4}{16}C_{23}^{0a}+C_{21}^{1T}\ln a+C_{22}^{1T} \quad (11g)$$

$$C_{21}^{1T}=-\frac{a^2}{2}C_{21}^{0a}+\frac{a^2}{4}(1-2\ln a)C_{22}^{0a}-\frac{a^4}{4}C_{23}^{0a} \quad (11h)$$

$$C_{22}^{1T}=-\frac{1}{4}\left(\frac{2}{Bi}+1\right)C_{21}^{0a}+\frac{1}{4}\left(\frac{1}{Bi}+1\right)C_{22}^{0a}-\frac{1}{16}\left(\frac{4}{Bi}+1\right)C_{23}^{0a}-C_{21}^{1T}. \quad (11i)$$

When $a=1$, the model will be reduced to a single layer. Hence, a comparison with the result of Xu and Davis [6] for single layer liquid bridge can be made, it is not difficult to get:

$$w_{01}=\frac{1}{2}\left(\frac{1}{2}-r^2\right)T'_a \quad (12a)$$

$$u_{01}=-\frac{1}{8}(r-r^3)T''_a \quad (12b)$$

and similarly,

$$w_{02}=\frac{1}{2}\left(\frac{1}{2}-r^2\right)T'_a \quad (12c)$$

$$u_{02}=-\frac{1}{8}(r-r^3)T''_a. \quad (12d)$$

COMPUTATIONAL RESULT OF AXIALLY HEATING

For the case where the liquid columns are heated at the left and cooled at the right, assuming a linear distribution of temperature for the gas, the radial velocity u_{01} of the inner layer and u_{02} of the outer vanish, whereas the axial velocity w_{01} and w_{02} are functions of parameters μ^*, γ and a. We particularly focused our attention on the average axial velocity $\bar{w}_{01}$ of the inner layer as defined by

$$\bar{w}_{01}=\frac{1}{a}\int_0^a|w_{01}|\,dr.$$

In order to compare with the result for a single layer, the scale variable of velocity must be unified for both cases, such that

$$w_*=\frac{\gamma_2\beta R_2}{\mu_2} \quad (13)$$

while the dimensionless average axial velocity for single layer should be expressed as

$$\bar{w}_s = \frac{\gamma^*}{\mu^*}\int_0^a \frac{1}{2}\left|\left(\frac{1}{2}-r^2\right)\right| dr. \tag{14}$$

We obtain the following calculating results:

(1) The axial average velocity of the inner layer (abbreviated by average velocity) decreases monotonously with the increasing viscosity ratio μ^*. The variation of parameters γ^* and a gives no change to such a tendency, but causes the value to be different, as shown in Fig. 2(a, b). Under conditions $\gamma^* = 1$, $a = 0.5$ and μ^* selected to be 0.1, 1, 10, respectively, the average velocity $\bar{w}_{01} = 0.027$, 0.013, 0.002 and $\bar{w}_s = 1.0417$, 0.10417, 0.010417, so the flow will be weakened by 97.5%, 87.5% and 80.8%, respectively, as compared with that for a single layer.

(2) The change of the average velocity $\bar{w}_{01}$ with γ^* for three assigned values of μ^* and a is shown in Fig. 3. It is seen that the average velocity decreases first until it reaches its minimum point, and then increases with γ^*. The value of γ^* at the minimum point shows a little change with μ^*,but it decreases with the increasing a. For $\mu^* = 1$ and $a = 0.5$, at the minimum point $\gamma^* \approx 0.6$, we have $\bar{w}_{01} = 0.0133$, the flow velocity reduces 92.4% as compared with $\bar{w}_s = 0.173616$.

(3) The change of average velocity with a at three assigned values of μ^* or γ^* is shown in Fig. 4. It is known from Fig. 4(a) that there are minimum and maximum points on the $\bar{w}_{01}$-a curve, whereas in Fig. 4(b) the minimum point will disappear as the absolute value of γ^* increases to a certain value. It seems that the influence of γ^* on $\bar{w}_{01}$ is rather strong.

Under conditions $\mu^* = 1$, $\gamma^* = 1$ and $a = 0.995$, the

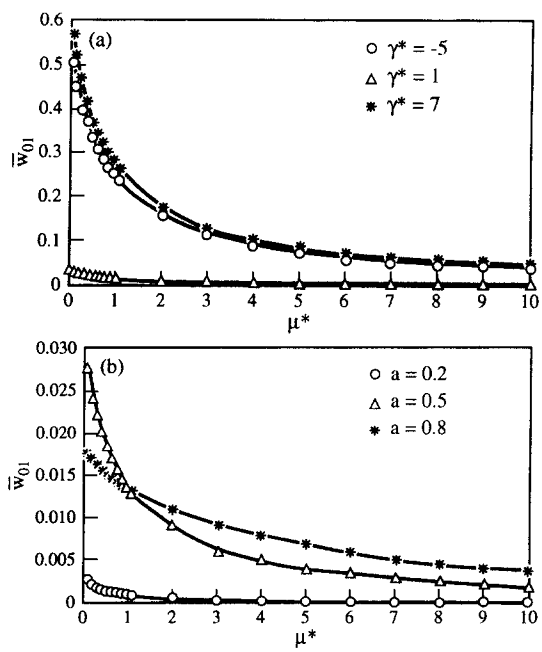

Fig. 2. (a) Variation of average axial velocity of inner layer with μ^* for $\gamma^* = -5,1,7$ and $a = 0.5$. (b) Variation of average axial velocity of inner layer with μ^* for $\gamma^* = 1$ and $a = 0.2, 0.5, 0.8$.

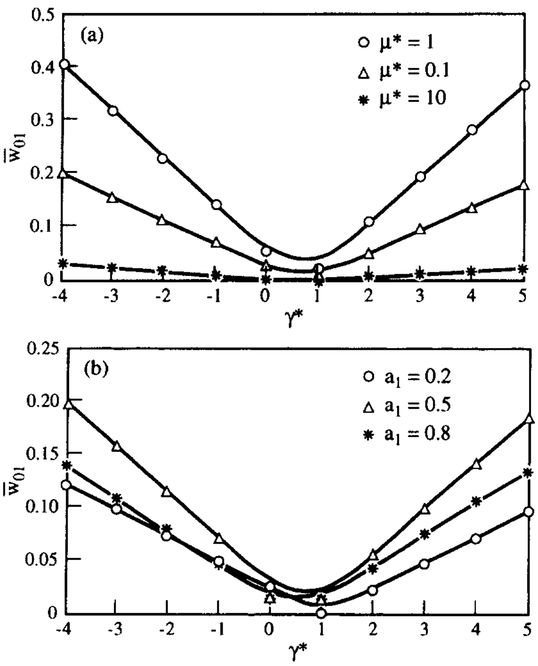

Fig. 3. (a) Variation of average axial velocity of inner layer with γ^* for $\mu^* = 0.1, 1, 10$ $a = 0.5$. (b) Variation of average axial velocity of inner layer with γ^* for $\mu^* = 1$ and $a = 0.2, 0.5, 0.8$.

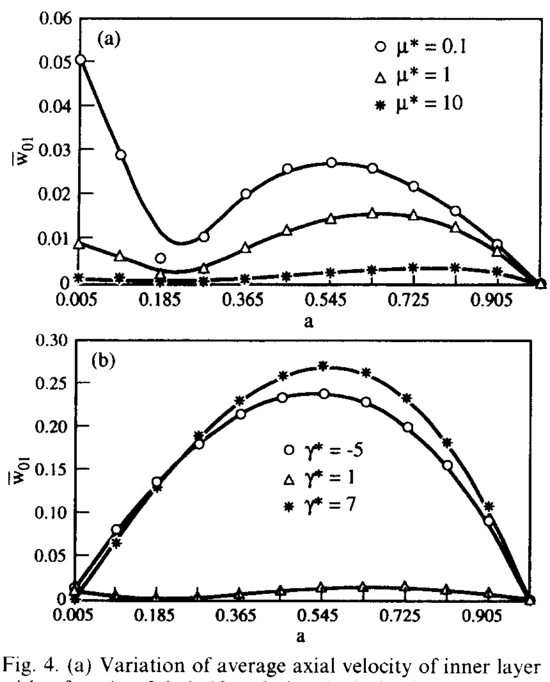

Fig. 4. (a) Variation of average axial velocity of inner layer with a for $\mu^* = 0.1, 1, 10$ and $\gamma^* = 1$. (b) Variation of average axial velocity of inner layer with a for $\mu^* = 1$ and $\gamma^* = -5, 1, 7$.

average velocity $\bar{w}_{01} = 0.0004645$, so the flow is weakened by 99.7% as compared with $\bar{w}_s = 0.1511314$.

(4) In Fig. 5 the distributions of the axial velocity in the vertical section going through the axle center

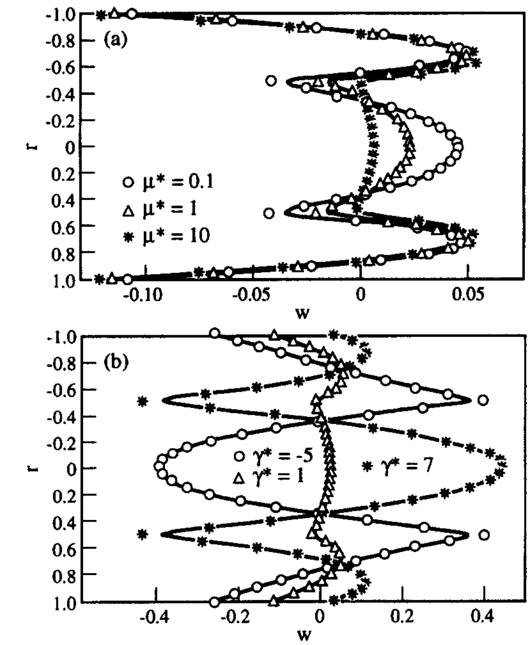

Fig. 5. (a) Axial velocity profile for $\mu^* = 0.1$, 1, 10, $\gamma^* = 1$ and $a = 0.5$. (b) Axial velocity profile for $\mu^* = 1$, $\gamma^* = -5$, 1, 10 and $a = 0.5$.

line are shown for the inner and outer layers, respectively. It is seen from the figures that the velocity at the interface decreases with increasing μ^* and the variation of the inner velocity tends to be smooth so that the flow is weakened. The liquid flow velocity at the free surface (interface 2), corresponding to its maximum value, increases slightly with increasing μ^*. The radial liquid velocity gradient near the free surface is not obviously affected by μ^*. It can also be seen from the figures that there are two cells (with regard to the upper half of the section) occurring in the outer layer when μ^* is small. By increasing the value of μ^* the cells near the interface become weaker and finally vanish, thus the flow turns to the single cell flow. From the maximum velocity on the curves, it seems that the affect of γ^* on the strength of the liquid flow is much stronger than parameter μ^*, and the value of γ^* will determine the number of cells in the outer layer (there is only one cell in the inner layer). Two cells occur when γ^* is positive and less than a certain value. The change of the sign of γ^* will result in the change of the flow direction. With regard to the upper half of the section, γ^* changes from negative to positive and will lead to a change of the flow direction of the fluid in the inner layer from clockwise to counter clockwise.

(5) Figure 6 shows the flow-induced temperature distribution. The curves of temperature distributions for $\gamma^* = \kappa^* = Bi = 2a = 1$ and selected values $\mu^* = 0.1$, 1, 10 are drawn in Fig. 6(a). The lowest temperature occurs at the axle center. The change of temperature in the inner layer along the radial direction tends to smooth as μ^* increases. Parameter μ^* shows a stronger effect on the temperature of the outer layer near the interface than onto the neighbour of the free surface 2, which is consistent with the flow field. The temperature distribution will become smooth as the absolute value of γ^* is small, corresponding to the weak flow case, whereas the tem-

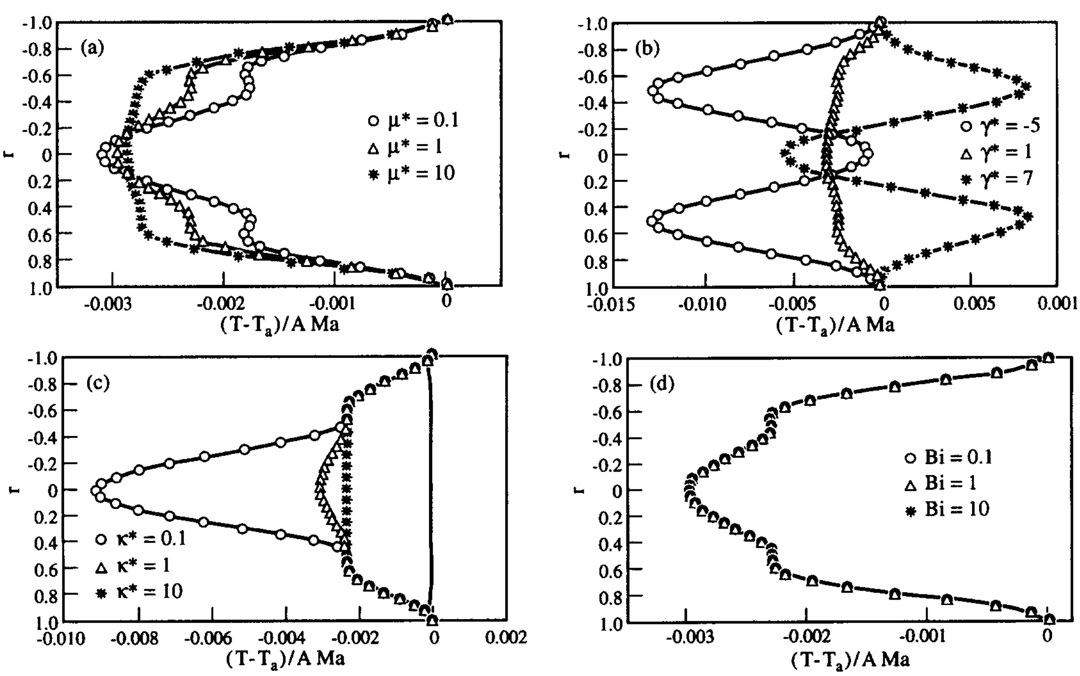

Fig. 6. (a) The profile of flow-induced temperature distribution for $\mu^* = 0.1$, 1, 10; $\gamma^* = \kappa^* = Bi = 2a = 1$. (b) The profile of flow-induced temperature distribution for $\gamma^* = -5$, 1, 7; $\mu^* = \kappa^* = Bi = 2a = 1$. (c) The profile of flow-induced temperature distribution for $\kappa^* = 0.1$, 1, 10; $\mu^* = \gamma^* = Bi = 2a = 1$. (d) The profile of flow-induced temperature distribution for $Bi = 0.1$, 1, 10; $\mu^* = \gamma^* = \kappa^* = 2a = 1$.

perature distribution shows an M-shape as the absolute value of γ^* getting larger. The extreme occur at the interface as well as the axle center, as shown in Fig. 6(b). Figure 6(c) shows the temperature distributions for $\kappa^* = 0.1$, 1, 10 and $\mu^* = \gamma^* = Bi = 2a = 1$. It is seen that the temperature distribution in the outer layer does not influenced by κ^*, whereas in the inner layer it increases with increasing κ^*, and the variation of temperature tends to smooth. Figure 6(d) shows the temperature distribution for $Bi = 0.1$, 1, 10 under condition $\mu^* = \gamma^* = \kappa^* = 2a = 1$. Bi has no affect on temperature distribution for both inner and outer layers.

CONCLUSIONS

(1) A mathematic model to describe the behaviour of the thermocapillary flow of two immiscible axisymmetric coaxial liquid columns is proposed as equations (5) and (6).

(2) An asymptotic solution valid for $A \to 0$ is obtained for the flow and thermal fields, emphasizing the dependence of radial and axial velocity and temperature distribution on property ratios of the two liquid layers, and the radius ratio of them.

(3) An axially heated example shows that effective reduction of the convection in the inner layer can be achieved through decreasing liquid viscosity of the outer layer, choosing a proper ratio of interface tension to free surface tension and thinning the encapsulation layer.

Acknowledgements—This work is supported by NSFC grant no. 59376261.

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